

Modeling and Forecasting the Crude Oil Price in Nigeria

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Abstract: This work is an attempt to examine empirically the best ARIMA and GARCH models for forecasting. The data employed in this study comprise of 189 monthly observations of crude oil price in Nigeria spanning from January, 1998 to September, 2013. At first the stationary condition of the data series are observed by autocorrelation function (ACF) and partial autocorrelation function (PACF) plots, then checked using Kwiatkowski–Phillips–Schmidt–Shin (KPSS) and Augmented Dickey Fuller (ADF) test statistic. It has been found that crude oil price is non-stationary. After we taking first difference of logarithmic values of data series, the same types of plots and the same types of statistics show that the data is stationary. The best ARIMA and GARCH models have been selected by using the criteria such as AIC, HQC, and SIC. The model for which the values of criteria are smallest is considered as the best model. Hence ARIMA (3, 1, 1) and GARCH (2, 1) are found as the best model for forecasting the crude oil price data series.

Keywords: crude oil price, Central Bank of Nigeria (CBN), Autocorrelation function (ACF), Partial Autocorrelation function (PACF).

1. INTRODUCTION

In recent years, crude oil price has become one of the major economic challenges facing most countries in the world especially those in Africa including Nigeria. Crude oil is a major focus of economic policy worldwide as described by Ayadi, O.F. (2005). Crude oil price dynamics and evolution can be studied using a stochastic modeling approach that captures the time dependent structure embedded in the time series crude oil price data. The Autoregressive Integrated Moving Average (ARIMA) popularly known as Box-Jenkins Methodology (G. P. E. Box and G. M. Jenkins (1978)) and the autoregressive conditional heteroscedasticity (ARCH) models, with its extension to generalized autoregressive conditional heteroscedasticity (GARCH) models as introduced by Engle (1982) and Bollerslev (1986) respectively accommodates the dynamics of conditional heteroscedasticity (the changing variance nature of the data). Heteroscedasticity affects the accuracy of forecast confidence limits and thus has to be handled properly by constructing appropriate non-constant variance models (Amos, 2010).

Oil plays a significant role in the Nigerian economy as the largest contributor in terms of total government revenue but also as the overall contributor in her exports composition. It accounted for about 82.1% of total government revenue during the oil boom in 1974 before reducing to a share of 64.3% by 1986 which was a consequence of the rapid decline in world market price of crude oil. The share of oil revenue in total government revenue still remains substantial as evidenced by the attainment of 85.6% and 86.1% in 2004 and 2005 respectively (Akpan,2009). The assumption of constant variance over some period when a series moves or progresses through time is statistically inefficient and inconsistent (Campbell et al, 1997). In real life, financial data variance changes with time (a phenomenon defined as heteroscedasticity), hence there is a need of studying models which accommodates this possible variation in variance. In considering the issue crude oil price modeling and forecasting in Nigeria, this work consequently intends to also use the Box-Jenkins methodology (ARIMA) and autoregressive conditional heteroscedasticity (ARCH) models with its extension to generalized ARCH (GARCH) models to model and accommodate the dynamics of conditional heteroscedasticity in

crude oil price data. Moreover by finding appropriate models to represent the data, the study intends to use them to predict future values based on the past observations.

Forecasting is an important part of econometric analysis, for some people probably the most important. How do we forecast economic variables, such as GDP, inflation, exchange rates, stock prices, unemployment rates, and myriad other economic variables? The method of forecasting that have become quite popular: Autoregressive Integrated Moving Average (ARIMA) popularly known as Box-Jenkins Methodology (G. P. E. Box and G. M. Jenkins (1978)), and the special problems involved in forecasting prices of financial assets, such as stock prices, exchange rates, etc. These assets prices are characterized by the phenomenon known as Volatility clustering, that is, periods in which they exhibit wide swings for an extended time period followed by a period of comparative tranquility. One only has to look at the Dow-Jones index in the recent past. The so-called Autoregressive Conditional Heteroscedasticity (ARCH) models can capture such volatility clustering. Philip Frances noted; "since such (financial time series) data reflect the result of trading among buyers and sellers at, for example, stock markets, various sources of news and other exogenous economic events may have an impact on the time series pattern of asset prices. Given that news can lead to various interpretations, and also given that specific economic events like an oil crisis cases can last for some time, we often observe that large positive and large negative observations in financial time series tend to appear in clusters." (Philip Hans Frances, (1998). Contreras et al. (2003) used ARIMA models to predict next day electricity prices; they have found two ARIMA models to predict hourly prices in the electricity markets of Spain & California. The Spanish model needs 5 hours to predict future prices as opposed to the 2 hours needed by the Californian model. Kumar et al. (2004) used ARIMA model to forecast daily maximum surface ozone concentrations in Brunei Darussalam. They have found that ARIMA (1, 0, 1) was suitable for the surface O₃ data collected at the airport in Brunei Darussalam. Tsitsika et al. (2007) used ARIMA model to forecast pelagic fish production. The final model selected were of the form ARIMA (1, 0, 1) & ARIMA (0, 1, 1). In his seminal 1982 paper, Robert F. Engle described a time series model with a time-varying volatility. Engle showed that this model, which he called ARCH (autoregressive conditionally heteroscedasticity), is well-suited for the description of economic and financial price. Nowadays ARCH has been replaced by more general and more sophisticated models, such as GARCH (generalized autoregressive heteroscedasticity). (Engle, (1982)).

2. AIM AND OBJECTIVES OF THE STUDY

The aim of this study is to establish models; using Box-Jenkins methodology and Volatility Models, to analyze crude oil price data based on Nigeria with a view to achieve the following objectives:

- To develop time series models (ARIMA and GARCH models) for the crude oil price data in Nigeria.
- To determine the accuracy of the models.
- To determine the future crude oil prices.

3. MATERIALS AND METHODS

The data employed in this study comprise of 189 monthly observations of the Crude Oil Price in Nigeria spanning from January, 1998 to September, 2013. Therefore the data generating process was subjected to Box-Jenkins ARIMA model. To apply the ARIMA tests the variable first examined for unit root and stationarity. In testing for the order of non stationary of the series Argument Dickey Fuller (ADF) (1979) and Kwiatkowski Phillips Schmidst and Shin (KPSS) (1992) test are used. If the series is non stationary then it is integrated indicating ARIMA as a model which result in good specification. The method used in this study is outlined below: Firstly, the data is presented graphically to check whether the data series is stationary or not. For this purpose, the statistics like Ljung-Box-Pierce Qstatistic (1978) based on auto correlation; Dickey-Fuller test (DF) (1979), Augmented Dickey-Fuller (ADF) test (1982) based on unit root process have been applied. These tests are discussed as follows:

Argument Dickey Fuller (ADF) test: Said and Dickey (1984) augment on the basic autoregressive unit root test to accommodate general ARMA (p, q) models with unknown orders and their test is referred to as the augmented Dickey-Fuller (ADF) test. The ADF test has the following hypothesis:

$$H_0 : \phi = 0 \quad \text{And } H_1 : \phi < 0.$$

The null hypothesis of a series is tested against the alternative hypothesis. And that the ADF test is based on following test regression

$$y_t = \beta' D_t + \phi y_{t-1} + \sum_{j=1}^p \psi_j \Delta y_{t-j} + \varepsilon_t \quad t = p+1, p+2, \dots, T \quad (1) \quad \text{Where } \beta' \text{ represents the trend in case it is}$$

present, ϕ is the coefficient of lagged dependent variable, y_{t-1} and p lags of Δy_{t-j} with a coefficient ψ_j are added to account for serial correlation in the residuals. The null hypothesis $H_0 : \phi = 0$ is that the series has unit root while the alternative hypothesis $H_1 : \phi \neq 0$ is that the series is stationary. The ADF t-statistic and normalized bias statistic are based on the least squares estimates and are given by

$$ADF_t = t_{\phi=1} = \frac{\hat{\Psi} - 1}{SE(\phi)}$$

$$ADF_n = \frac{T(\hat{\phi} - 1)}{1 - \hat{\Psi}_1 - \dots - \hat{\Psi}_p}$$

Where $SE(\phi)$ is the standard error for (ϕ) and denotes estimate. The null hypothesis of unit root is accepted if the test statistics is greater than the critical values. A useful rule of thumb for determining P_{\max} , suggested by Schwert (1989), is

$$P_{\max} = \left[12 \cdot \left(\frac{T}{100} \right) \right]^{1/4}$$

Kwiatkowski Phillips Schmidt and Shin: The most commonly used stationarity test, the KPSS test, is due to Kwiatkowski, Phillips, Schmidt and Shin (1992). The integration properties of a y_t may also be investigated by testing the null hypothesis that the series is stationary against a unit root. Kwiatkowski et al. (1992) derived a test for this pair of hypothesis. Assuming no linear trend term, the data generating process is given as

$$y_t = x_t + z_t \quad (2)$$

Where x_t a random is walk, $x_t = x_{t-1} + v_t, v_t \square iid(0, \sigma_v^2)$ and z_t is a stationary process. KPSS process the following test statistics.

$$T^{-2} \sum_{i=1}^n \frac{S_i^2}{\hat{\sigma}_\infty^2} \quad (3)$$

Where $S_t = \sum_{i=1}^n \hat{X}_j$ with $w_j = y_j - \bar{y}$ and $\hat{\sigma}_x^2$ an estimator of the long run variance of

$z_t, \sigma_\infty^2 = \lim_{T \rightarrow \infty} T^{-1} \text{var} \left[\sum_{t=1}^T z_t \right]$ The null hypothesis of the test is $H_0 : \sigma_v^2 = 0$ against alternative hypothesis

$H_1 : \sigma_v^2 \neq 0$. This test uses the Bartlett window with a lag truncation parameter

$$I_q = q \left[\frac{T}{100} \right]^{1/4} \quad 4$$

Where

$$\hat{\sigma}_v^2 = T^{-1} \sum_{t=1}^T \hat{w}_t^2 + 2 \sum_{j=1}^i w_j \left[T^{-1} \sum_{t=j+1}^T \hat{w}_t \hat{w}_{t-j} \right]$$

And

$$w_j = 1 - \frac{j}{I_4 + 1}$$

Reject the null hypothesis if the test statistics is greater than the asymptotic critical values.

To select the best ARIMA (p, d, q) type of models fitted for the company, their goodness of fit have been compared using following criteria;

Akaike Information Criterion (AIC): AIC is an important and leading statistics by which we can determine the order of an autoregressive model Mr. Akaike developed this statistics. According to his name this statistics is known as Akaike Information Criterion (AIC). The AIC takes into account both how well the model fits the observed series and the number of parameters to be used in the fit. AIC due to Akaiken (1969) is defined as

$$AIC = N(\hat{\delta}^2 + 1) + 2(p + 1) \quad 5$$

Where the parameter bears the usual meaning. Akaike also mention that the minimum AIC criterion produced a selected model, which is hopefully closer to the best possible choice.

Schwartz Information Criteria (SIC): In 1978 Schwartz discussed a criterion denoted by SIC which help in deciding the order of auto regression. Initially he developed this criterion for taking decisions about the regress subset. Later Engel et. al, in 1992 use this criterion as a tool for determining the order of auto regression and they defined this criterion as below:

$$SIC = \hat{\delta} \left(-\frac{p}{N} \right)^{\frac{1}{2}} N^{\frac{p}{2N}} \quad 6$$

Where, the parameters bear the usual meaning. Schwartz also shows that this criterion is better than AIC.

The model with minimum SIC assumes to describe the data series adequately. The minimum value of this criterion is desirable for the adequacy of a model.

Autoregressive Integrated Moving Average ARIMA (P,d,q) model: In general, an ARIMA model is characterized by the notation $(ARIMA_{(p,d,q)})$ where p, d and q denote orders of autoregressive, integration (differencing) and moving average respectively. In ARIMA parlance, time series is a linear function of past actual values and random shocks.

A time series X_t which is integrated of order d has the ARIMA if it is represented in the form:

$$\phi(L)(1-L)^d X_t = \mu + \theta(L)\varepsilon_t, \dots, \dots, 4_7$$

Where $(1-L)^d$ is the integrated of order d . where ε_t is independently and normally distributed with zero mean and constant variance. Where L denotes the lag operator.

Tool: There are many statistical software's used in time series analysis depending on one's choice or features. In this research work the statistical software used is Gretl 1.8.0 which is based on c programming language. Gretl is an open

statistical package mainly for econometric. The name is an acronym for Gnu regression, econometric and time series analysis. It uses the gnu plot to generate the graph.

Residual analysis: In the test for residual there are various test including portmanteus test, lagrange multiplier, autocorrelation and partial autocorrelations, Jarque-Bera test, and Ljung –Box test, all test for the adequacy of the model.

Autoregressive conditional heteroscedasticity: The ARCH process introduced by Engle (1982) and Tim Bollerslev (1986) explicitly recognized the difference between the unconditional and the conditional variance allowing the latter to change over time as a function of past errors.

Let ε_t denote a real-valued discrete –time stochastic process and Ψ_t the information set ($\sigma - field$) of all information through time t. then

$$\varepsilon_t / \Psi_{t-1} \square N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}$$

$$= \alpha_0 + A(L)\varepsilon_t^2 + (L)h_t,$$

Where

$$p \geq 0, \quad q > 0$$

$$\alpha_0 > 0, \quad \alpha_i \geq 0 \quad i = 1, \dots, q$$

$$\beta_i \geq 0, \quad i = 1, \dots, p$$

For p=0the process reduces to the ARCH(q) process and for p=q=0 is simply white noise.

A Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model: If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a generalized autoregressive conditional heteroscedasticity (GARCH) model, Bollerslev (1986). In that case, the GARCH (p, q) model (where p is the order of the GARCH terms σ^2 and q is the order of the ARCH terms e^2) is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \dots + \alpha_q e_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_p \sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^q \alpha_i e_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \quad 8$$

Generally, when testing for heteroskedasticity in econometric models, the best test is the white noise test. However, when dealing with time series data, this means to test for ARCH errors (as described above) and GARCH errors (below).

GARCH (p, q) model specification: The lag length p of a GARCH (p, q) process is established in three steps:

1. Estimate the best fitting AR (q) model

$$y_t = a_0 + a_1 y_{t-1} + \dots + a_q y_{t-q} + e_t = a_0 + \sum_{i=1}^q a_i y_{t-i} + e_t \quad 9$$

2. Compute and plot the autocorrelations of e^2 by

$$\rho = \frac{\sum_{t=i+1}^T (\hat{e}_t^2 - \hat{\sigma}_t^2)(\hat{e}_{t-1}^2 - \hat{\sigma}_{t-1}^2)}{\sum_{t=1}^T (\hat{e}_t^2 - \hat{\sigma}_t^2)^2} \quad 10$$

3. To estimate the total number of lags, use the Ljung-Box test until the value of these are less than, say, 5% significant. The Ljung-Box Q-statistic follows χ^2 distribution with n degrees of freedom if the squared residuals e_t^2 are uncorrelated. It is recommended to consider up to $T/4$ values of n . The null hypothesis states that there are no ARCH or GARCH errors. Rejecting the null thus means that such errors exist in the conditional variance.

4. ANALYSIS OF DATA

Initial analysis of data: The time plot in fig 1 shows the original series. The time plot of the series gives an initial step about the likely nature of the crude oil price time series. The series plot exhibit the phenomenon of volatility clustering, that is, periods (in which the oil price) shows wide swings for an extended time period followed by periods in which there is relative calm. Over the period of study, the crude oil price has been increasing, that is, showing an upward trend, in a fluctuational pattern, suggesting perhaps that the mean and the variance of the log of crude oil price has been changing with time or over time in Nigeria. The fig 2 is the correlogram (ACF and PACF) of the crude oil price's log of the data series before differencing. The most striking feature of this correlogram is that the autocorrelation coefficients at various lags are very high, (at lag 1 = 0.9777) up to a lag of 47 months (at lag 47 = 0.2695); these are individually statistically significantly different from zero, out of the 95% confidence bounds. This is the typical correlogram of a non-stationary time series. The autocorrelation starts at a very high value and a decline (spikes down) very slowly toward zero as the lags lengthens, showing a purely MA series. After the first three lags, the PACF drops dramatically, and most PACFs after lag 3 are statistically insignificant, showing an AR of order 3. We conclude from the result of the ACF that there is need for differencing which indicate the series is an ARIMA process.

Unit root tests before differencing: Two different tests were conducted Augment Dickey Fuller (ADF) test and Kwiatkowski Phillips Schmidst and Shin (KPSS) test. The results of the test are given in table 1 which show that the confirming the log of the crude oil price series is not stationary, since the tests statistics are greater than the p/critical values at some levels. Since the time series is not stationary, we have to make it stationary before we can apply the Box-Jenkins methodology. This can be done by differencing the series once. Fig 3 shows the time plot of the first difference. A visual inspection of the plot show the series has constant mean and variance. The plot however, does not show any evidence of stationary. The correlogram is shown in fig 4: The ACFs at lag 1, 2, 6, 8, and 29 seem statistically different from zero (at the 95% confidence limit, those lags are asymptotic and so can be considered approximate), but at all other lags, they are not statistically different from zero. We therefore conclude that the data series is now stationary. A formal application of the Augmented Dickey-Fuller and KPSS unit root tests below may show that this is indeed the case.

Unit root tests after differencing: Two different tests were conducted Augment Dickey Fuller (ADF) test and Kwiatkowski Phillips Schmidst and Shin (KPSS) test. The results of the test are given in table 2 which show the entire test statistics of the ADFs are less than the critical regions, we reject the null hypothesis and therefore conclude that there is no unit root or the time series is stationary. Likewise the KPSS tests, the test statistics are all less than the p-values; we therefore accept the null hypothesis and conclude that the data series is stationary around a deterministic trend.

Formulation of the ARIMA model for the crude oil price data: from the initial analysis (time plot and correlogram of the data) it is clearly shown that the series is non stationary. This led to the first differencing $d=1$.

Model Identification and selection: The table 3 tested fifteen (15) models with low AIC, HQC and SIC which is common in ARIMA modeling and find the best models among them. ARIMA (3, 1, 1) model is selected because they have minimum AIC, HQC and SIC. The estimates of the parameters of the model, shown in table 4, indicates that AR (1), AR (3) and MA (1) models are significant at the 0.05 significance level while AR (2) is insignificant at 0.05 level of significance. Our diagnostic checking of the GARCH (2, 1)

Model checking for ARIMA (3, 1, 1): Following McLeod A.I and Li W.K (1983), the best model is next tested for adequacy using a diagnostic test that is ARCH-LM test. The result is shown in Table 5. The diagnostic test result shows that the model has passed the diagnostic test of normality. The test show there is no ARCH-LM effect present; hence the residuals have a constant variance. In fig. 5: The ACF and PACF of ARIMA (3,1,1) model of the residuals also show that the residuals are white noise series.

ARIMA (3, 1, 1) Forecast Evaluation: After a good ARIMA model has been fitted, we finally study its forecast value. Table 6 contains six months samples forecast while fig 6 shows the graph. The prediction is similar to the observed value in pattern; this testified the adequacy of our model. Although the forecast sample is better.

Formulation of the GARCH model and Analysis for the crude oil price data:

Model Identification and selection: The table 7 tested six (6) models with low AIC, HQC and SIC which is common in GARCH modeling and find the best models among them. GARCH (2, 1) model is selected because they have the minimum AIC, HQC and SIC. The estimates of the parameters of the model, shown in table 7, indicates that AR (0), AR (1) and MA (1) models are significant at the 0.05 significance level while AR (2) is significant at 0.10 level of significance.

Model checking for GARCH (2, 1): In the pre-estimation analysis, the ARCH test indicated rejection of the null hypothesis showing significant evidence in support of GARCH effects. The residual tests below show that no any ARCH effects left (no heteroskedasticity). Also from the test result in table 8, since the p-values of 0.153 (Q-statistic), 0.216616 (ARCH-LM) is greater than 5% alpha level, we fail to reject the null hypothesis that, there is no autocorrelation left in the residuals. Therefore we proceed to use the models to forecast future values of the Crude Oil Prices series. In fig. 7: The ACF and PACF of GARCH (2, 1) model of the residuals also show that the residuals are white noise series.

GARCH (2, 1) Forecast Evaluation: After a good GARCH model has been fitted, we finally study its forecast value. Table 9 contains six months samples forecast while fig 8 shows the graph. It can be confirmed that the forecasted values are close to the actual values, thus, the model somehow fit the data well.

5. CONCLUSION

This study made the best endeavor to develop the best ARIMA and GARCH models to efficiently forecasting the crude oil price. After which the data was subjected to various test to choose the best model. The empirical analysis indicated that the ARIMA (3, 1, 1) and GARCH (2, 1) models are best for forecasting the crude oil price data series so far the diagnostic criteria are concerned. It now used in forecasting for six month. We concluded that prediction from our forecast shows a drastic increase in the crude oil price when it is compared to the previous records.

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APPENDIX - A

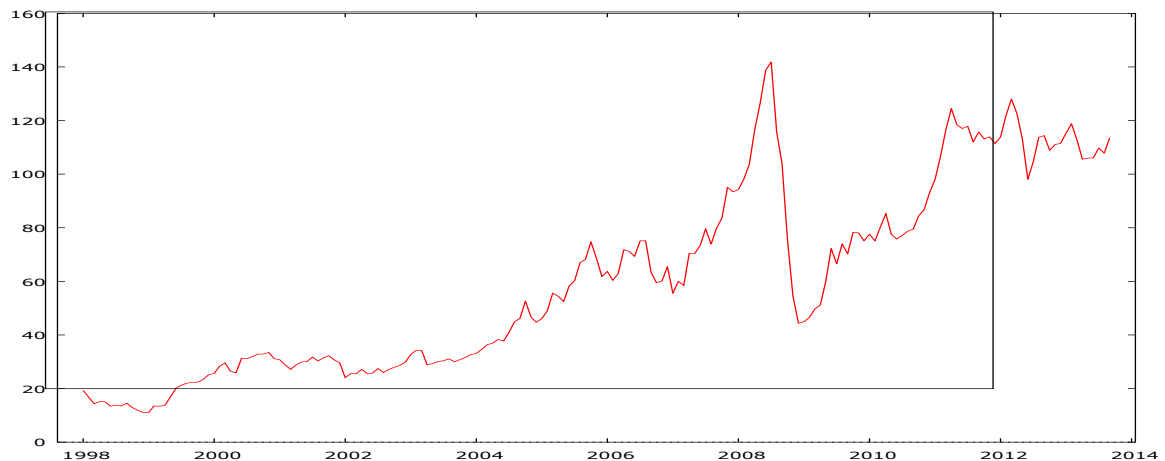


Fig. 1: Time plot of the monthly crude oil price

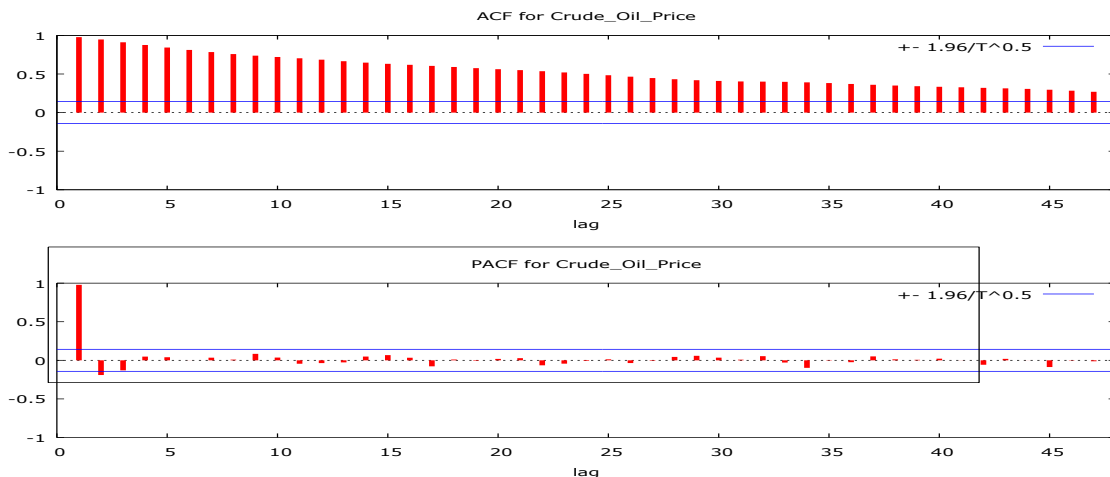


Fig. 2: The ACF and PACF of the monthly crude oil price

UNIT ROOT TESTS BEFORE DIFFERENCING

Table 1: ADF and KPSS tests

TEST	TEST STATISTIC	CRITICAL/P-VALUES		
ADF without constant	1.55578	0.9711		
ADF with constant	-0.759884	0.8296		
ADF with constant and trend	-1.47992	0.8367		
KPSS without trend	0.519286	1%	5%	10%
		0.739	0.463	0.347
KPSS with trend	0.135475	1%	5%	10%
		0.216	0.146	0.119

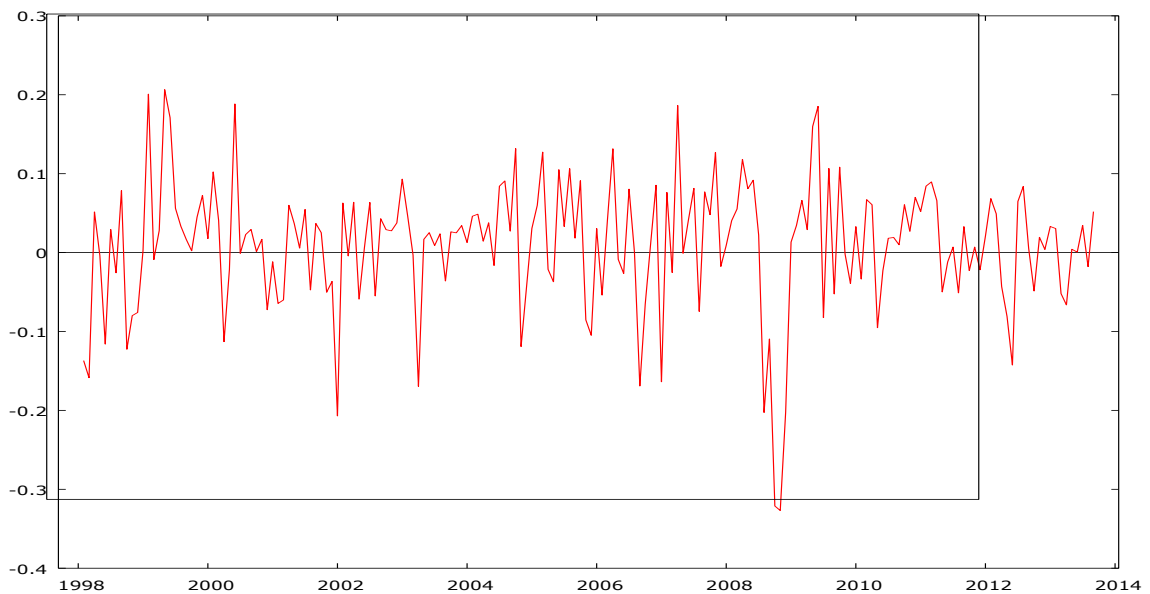


Fig. 3: The first difference of crude oil price

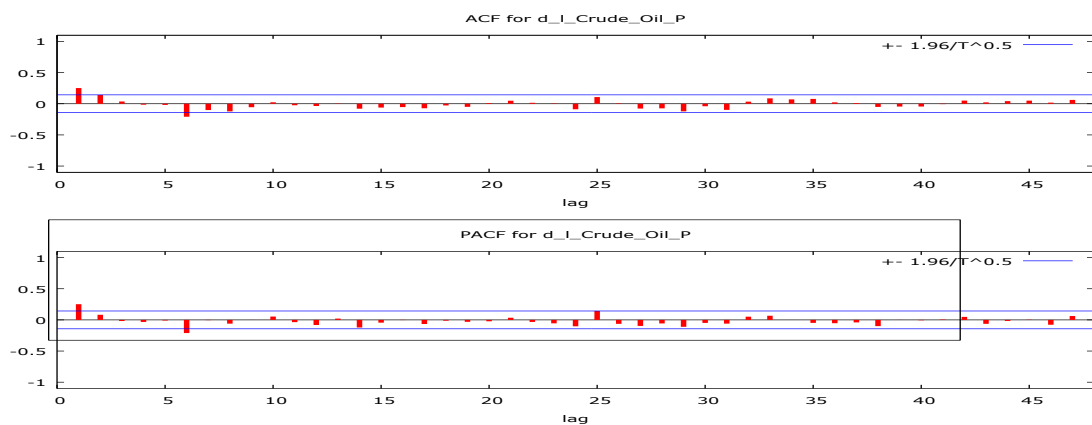


Fig. 4: ACF and PACF after first differencing

UNIT ROOT TESTS AFTER DIFFERENCING

Table 2: ADF and KPSS tests

TEST	TEST STATISTIC	CRITICAL/P-VALUES		
ADF without constant	-1.0677	0.2588		
ADF with constant	-2.05021	0.2654		
ADF with constant and trend	-2.19248	0.4933		
KPSS without trend	0.141688	1%	5%	10%
		0.739	0.463	0.347
KPSS with trend	0.133236	1%	5%	10%
		0.216	0.146	0.119

Table 3: Result of ARIMA model identification and selection

MODEL	AIC	HQC	SIC
ARIMA(0,1,1)	-409.5219	-405.5881	-389.8126
ARIMA(0,1,2)	-411.2306	-405.9854	-393.2848
ARIMA(0,1,3)	-409.3362	-402.7781	-393.1540
ARIMA(1,1,0)	-411.8455	-407.9116	-302.1361
ARIMA(1,1,1)	-410.9250	-405.6799	-397.9793
ARIMA(1,1,2)	-409.3839	-402.8275	-397.9793
ARIMA(1,1,3)	-407.3873	-399.5196	-387.9687
ARIMA(2,1,0)	-411.2728	-406.0276	-393.3270
ARIMA(2,1,1)	-409.3158	-402.7594	-393.1336
ARIMA(2,1,2)	-407.3889	-399.5212	-387.9687
ARIMA(2,1,3)	-415.6789	-406.4999	-396.0238
ARIMA(3,1,0)	-409.3531	-402.7966	-393.1709
ARIMA(3,1,1)	-417.1257	-409.2580	-397.7070
ARIMA(3,1,2)	-415.6794	-406.5004	-397.0243
ARIMA(3,1,3)	-408.4851	-397.9948	-382.5935

Table 4: Result of ARIMA (3, 1, 1) model estimation

PARAMETER	COEFFICIENT	STD. ERROR	T-RATIO	P-VALUE
AR (1)	1.17612	0.0721479	16.30	9.62e-060
AR (2)	-0.117493	0.111749	-1.051	0.2931
AR (3)	-0.144863	0.0730737	-1.982	0.0474
MA (1)	-1.00000	0.0164318	-60.86	0.0000

Table 5 ARCH-LM test of ARIMA (3, 1, 1) model

TEST	TEST STATISTIC	P-VALUE
ARCH-LM	31.7143	0.957

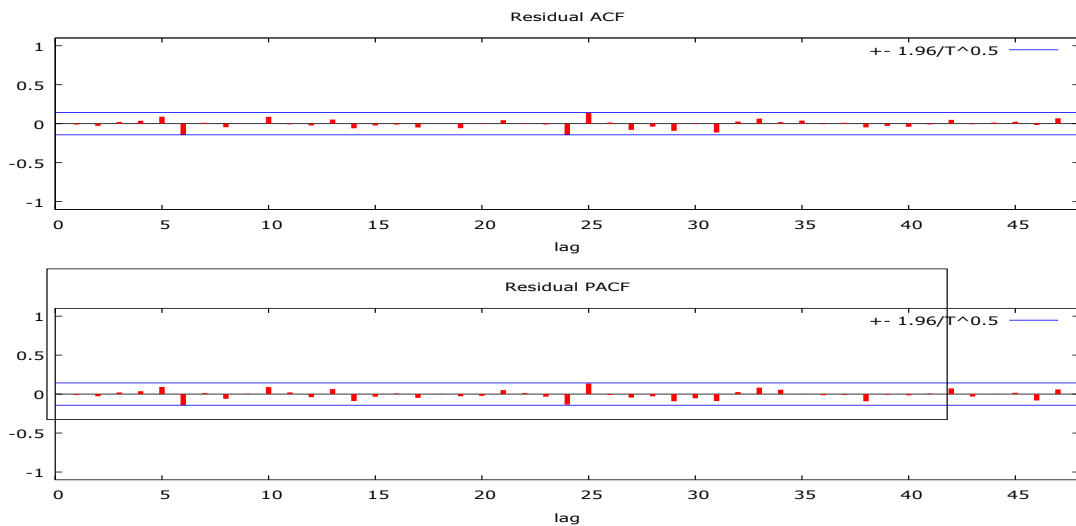


Figure 5: The Residual ACF and PACF of ARIMA(3,1,1) model

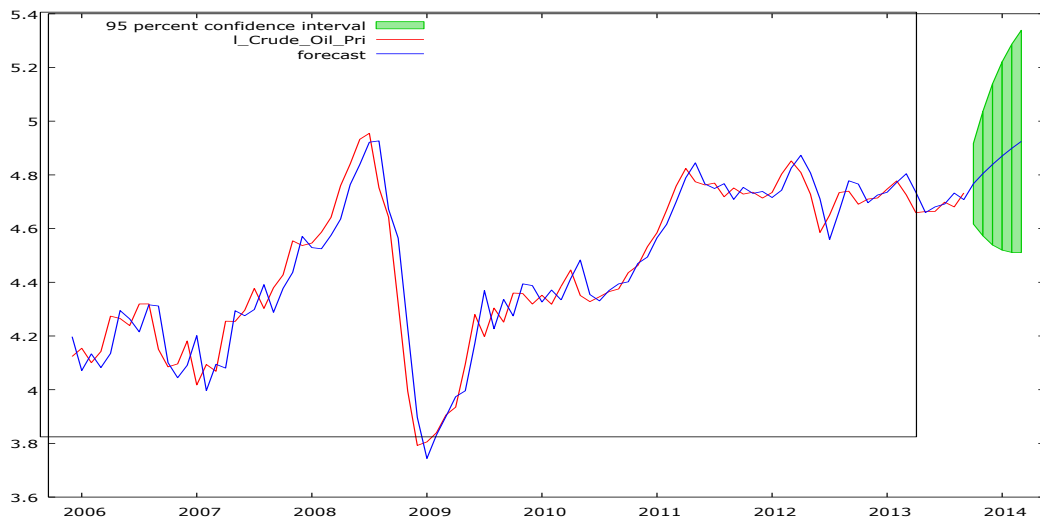


Figure 6: ARIMA (3, 1, 1) plot of 6 Months Forecasted result

Table 6: Result of ARIMA (3, 1, 1) 6 Months forecasted values (for 95% CI, $z(0.025) = 1.96$)

Observations	Prediction	std. error	95% confidence interval
2013:10	4.76705	0.0767084	(4.61670, 4.91740)
2013:11	4.80499	0.118421	(4.57288, 5.03709)
2013:12	4.83898	0.153137	(4.53883, 5.13913)
2014:01	4.87045	0.178897	(4.51981, 5.22109)
2014:02	4.89892	0.197762	(4.51131, 5.28653)
2014:03	4.92472	0.211227	(4.51072, 5.33873)

Table 7: Parameter Estimation for GARCH (2, 1)

PARAMETER	COEFFICIENT	STD. ERROR	T-RATIO	P-VALUE
alpha(0)	0.689748	0.323507	2.132	0.0330
alpha(1)	0.155002	0.100767	2.538	0.0240
alpha(2)	0.232940	0.143212	1.627	0.1038
beta(1)	0.612058	0.0886099	6.907	4.94e-012

Table 8: Summary of the Result of GARCH (2,1) model checking

TEST	TEST STATISTICS	P-VALUE
Box-Pierce Q-Statistic	56.8717	0.153
ARCH-LM	94.6772	0.216616

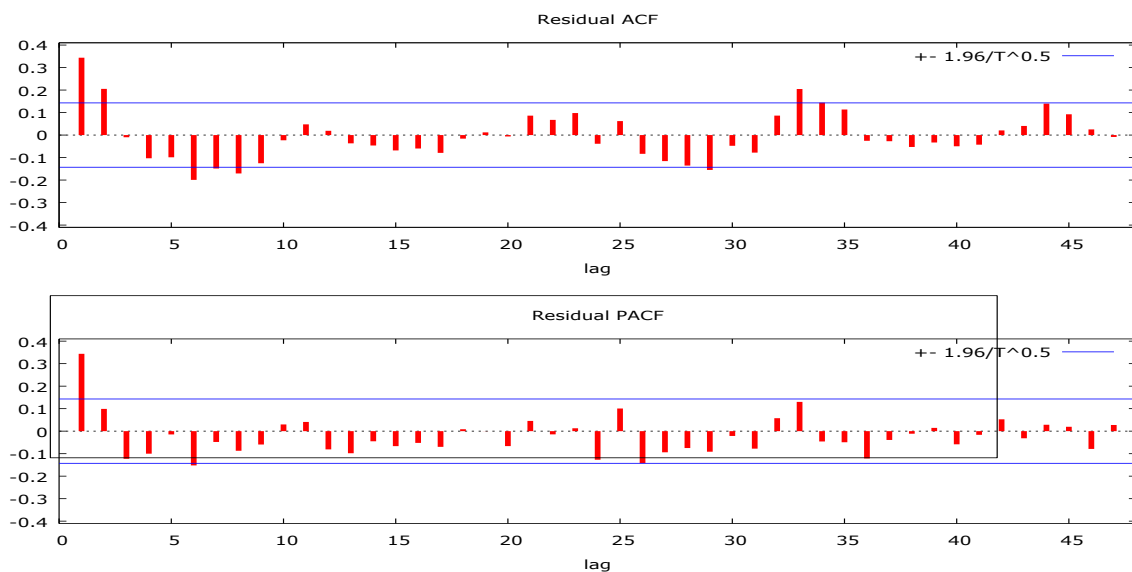


Figure 7: The Residual ACF and PACF of GARCH(2,1) model

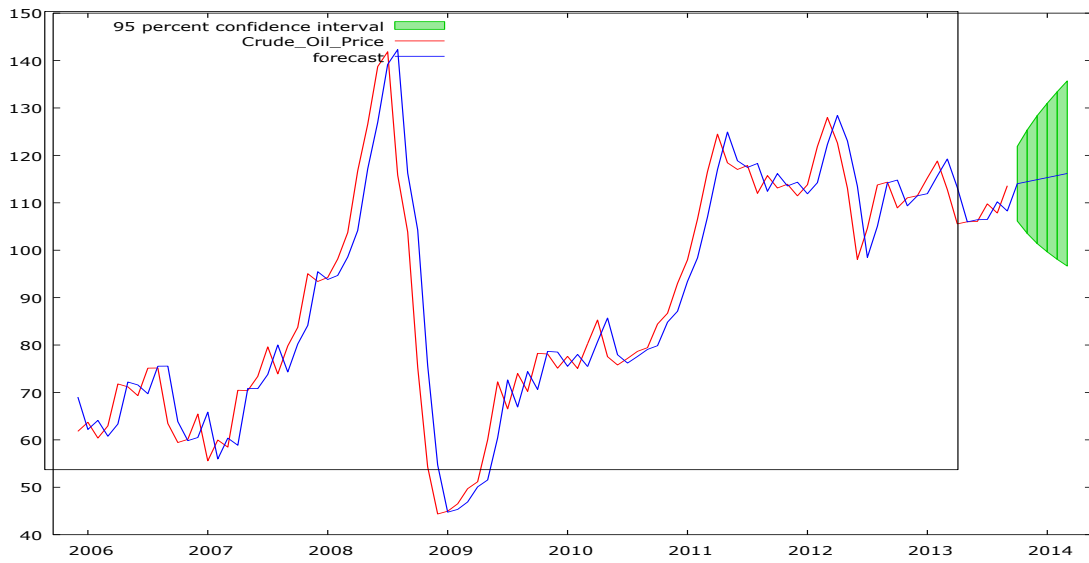


Figure 8: GARCH (2, 1) plot of 6 Months Forecasted result

Table 9: Result of GARCH (2,1) 6 Months forecasted values (for 95% CI, $z(0.025) = 1.96$)

Observations	Prediction	std. error	95% confidence interval
2013:10	114.021	3.98955	(106.151, 121.892)
2013:11	114.452	5.53364	(103.536, 125.369)
2013:12	114.884	6.84391	(101.383, 128.386)
2014:01	115.316	7.94838	(99.6356, 130.997)
2014:02	115.748	8.96374	(98.0648, 133.432)
2014:03	116.181	9.89899	(96.6523, 135.710)